

Observation of two coupled Faraday waves in a vertically vibrating Hele-Shaw cell with one of them oscillating horizontally

Xiaochen Li, Xiaoming Li and Shijun Liao *

State Key Laboratory of Ocean Engineering
School of Naval Architecture, Ocean and Civil Engineering
Shanghai Jiaotong University, Shanghai 200240, China

Abstract *A system of two-dimensional, two coupled Faraday interfacial waves is experimentally observed at the two interfaces of the three layers of fluids (air, pure ethanol and silicon oil) in a sealed Hele-Shaw cell with periodic vertical vibration. The upper and lower Faraday waves coexist: the upper vibrates vertically, but the crests of the lower one oscillate horizontally with unchanged wave height and a frequency equal to the half of the forcing one of the vertically vibrating basin, while the troughs of the lower one always stay in the same place (relative to the basin). Besides, they are strongly coupled: the wave height of the lower Faraday wave is either a linear function (in the case of a fixed forcing frequency) or a parabolic function (in the case of a fixed acceleration amplitude) of that of the upper, with the same wave length. In addition, the upper Faraday wave temporarily loses its smoothness at around $t = T/4$ and $t = 3T/4$, where T denotes the wave period, and thus has fundamental difference from the traditional one. To the best of our knowledge, this system of the two coupled Faraday waves has never been reported.*

Key Words Faraday waves; multiple layers of fluids; experimental observation

1. Introduction

The Faraday waves in a vertically oscillating basin were first discovered by Faraday [1] and then analyzed by Benjamin and Ursell [2], who found that these standing waves vertically vibrate with a frequency equal to half of the forcing one of the basin. These waves can organize in different forms,

*Corresponding author. Email address: sjliao@sjtu.edu.cn

such as stripes, squares, hexagons [3], and even stars [4]. The Faraday instability in viscous fluids was also experimentally investigated by Bouchgl and Aniss [5]. Thereafter, the motion of the interface between two fluids with different ratios of density by means of forcing vertical oscillation became a hot topic. Some extreme steep interfacial waves which oscillate vertically at the interface of two inviscid fluids were numerically simulated and their stability was investigated by Mercer and Roberts [6]. The parametric instability analysis for the interface of two viscous fluids was studied by Kumar and Tuckerman [7], who found that the effect of large viscosity on the wavelength selection is substantial. The two-dimensional Faraday waves of two inviscid fluids were numerically studied by Wright et al. [8], the results were also compared with the fully nonlinear numerical simulation by Takagi and Matusumoto [9]. The instability of Faraday interfacial waves between two weakly viscous layers in a rectangular domain was studied by Hill [10]. The spatiotemporal Fourier spectrum of Faraday waves on the interface of two liquids in a three-dimensional closed cell were measured by Kityk [11]. The experimental study on Faraday waves in domains with flexible boundaries is implemented by Pucci et al. [12, 13] in the instability of floating fluid drops. The walking and orbiting droplets were observed on the surface of a liquid at a sufficiently high acceleration by Couder et al. [14]. The linear Faraday stability of a two-layer liquid film with a free upper surface was investigated numerically by Potosky and Bestehorn [15]. The diffuse interface between two miscible liquids subject to vertical vibration was studied by means of experiments and numerical simulation [16], and a time-dependent density gradient is established from the moment when the two layers were placed together [17]. By singular perturbation theory, the interfacial wave modes in a two-layer liquid-filled cylindrical vessel were found to become more complex, as the density ratio increases from the upper to the lower layer [18].

In this letter we experimentally investigate the system of the two coupled interfacial Faraday waves at the interfaces of air and two immiscible liquids in a sealed Hele-Shaw cell with periodic vertical vibration. The upper liquid is pure ethanol (with the density $\rho_1 = 791\text{kg/m}^3$ and the viscosity $\mu_1 = 0.0011\text{Pa s}$) and the lower is silicon oil (methyl-silicone-I, with the density $\rho_2 = 970\text{ kg/m}^3$ and the viscosity $\mu_2 = 0.35\text{ Pa s}$). Above the two immiscible liquids is the air. So, there exist three layers of different fluids and two interfaces. One is the interface between the air and the pure ethanol, called the upper interface. The other is the interface between the pure ethanol and the silicon oil, called the lower interface.

2. Experimental setup

The experimental setup is as follows. A Hele-Shaw cell (made of PMMA) with 300 mm length, 2mm width and 60mm depth is filled with two immiscible fluids: the upper is pure ethanol (4mm in depth) and the lower is silicone oil (8mm in depth). For the sake of observation convenience, a very small amount of phenol red is added in pure ethanol. The cell is fixed on a horizontal shaker and guided with a vertical sinusoidal vibration. The forcing frequency (denoted by f) and the acceleration amplitude (denoted by A) of the shaker are output by a closed-loop control system. A high-speed camera is positioned perpendicular to the front of the cell to record the evolution of the upper interface (between the air and pure ethanol) and the lower interface (between the two liquids). The temperature is nearly 20°C. Considering volatility of pure ethanol, the cell is sealed (i.e. the depth of the air is 48 mm) and the pure ethanol is replaced every twenty minutes.

3. Experimental results

When the Hele-Shaw cell vibrates vertically with the forcing frequency $f=18$ Hz and the acceleration amplitude $A=17$ m/s², we observed a system of two coupled Faraday waves. For details, please see figure 1 and the corresponding movie. At the upper interface, there exists a standing wave that oscillates *vertically* in a similar way like a traditional Faraday wave (but with a few fundamental differences mentioned later), call the upper Faraday wave. At the lower interface, there exists a standing wave whose crests oscillate *horizontally* with an unchanged height and a frequency equal to half of the forcing frequency of the *vertically* vibrating basin, called the lower Faraday wave. To the best of our knowledge, such kind of horizontally oscillating Faraday waves have never been reported. The upper and lower Faraday waves coexist and are strongly coupled, with the same period (denoted by T) and the same wave length (denoted by L). At $t = 0$, the crest of the upper Faraday wave reaches its maximum height, below which there are two adjoining crests of the lower Faraday wave that are in the shortest distance (denoted by δ_{min}). Thereafter, the crest of the upper Faraday wave falls vertically until it becomes a trough at $t = T/2$, while the above-mentioned two adjoining crests of the lower Faraday wave depart horizontally from each other with almost unchanged height, and their distance (denoted by δ) increases until it reaches the maximum (denoted by δ_{max}) at $t = T/2$. As the time further

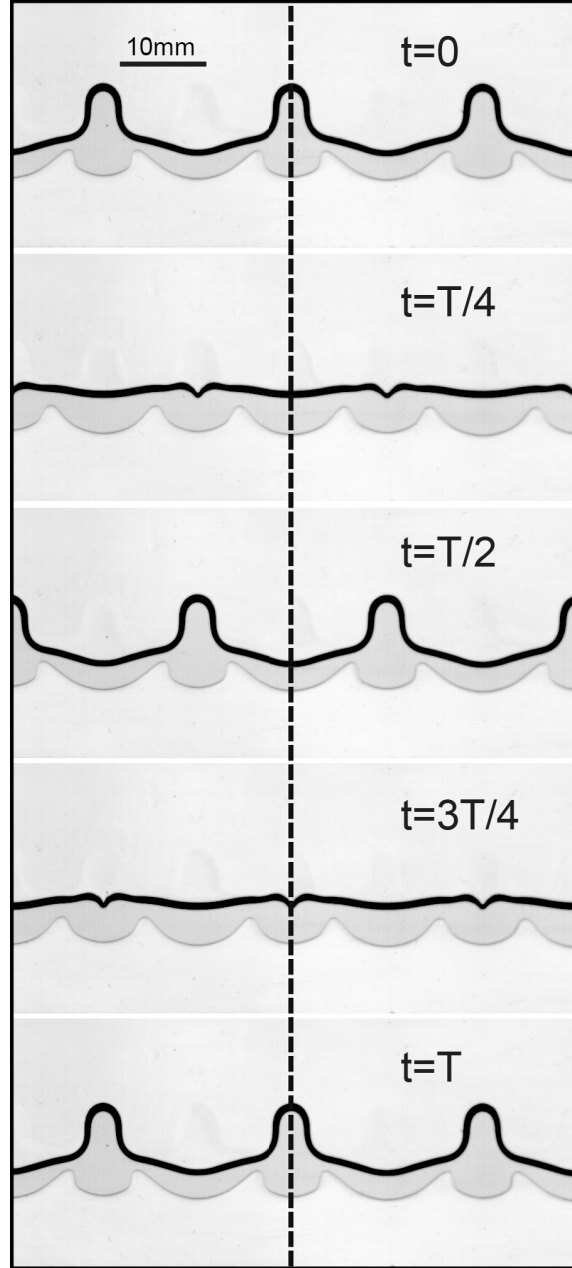


Figure 1: (Colour online) The system of two coupled Faraday waves in the case of the forcing frequency $f=18$ Hz and the acceleration amplitude $A = 17 \text{ m/s}^2$, where T denotes the wave period. For more details, please see the corresponding movie.

increases, the trough of the upper Faraday wave moves upwards, but temporarily loses its smoothness at $t = 3T/4$, and then becomes a crest again that reaches its maximum at $t = T$. In the same time, the two adjoining crests of the lower Faraday wave horizontally approach each other with the unchanged height until δ decreases to δ_{min} at $t = T$. Note that all troughs of the lower Faraday wave are still (relative to the Hele-Shaw cell) on the same horizontal line, and the distance of any two adjoining troughs is equal to the half of the wave length L of the upper Faraday wave. However, unlike the upper Faraday wave which has a symmetry about crest, the lower Faraday wave loses its symmetry about the crest, although both of the upper and lower Faraday waves retain the symmetry about the trough. Besides, unlike the traditional Faraday wave, the upper Faraday wave temporarily loses its smoothness at around $t = T/4$ and $t = 3T/4$. It implies that the upper and lower Faraday waves strongly interact each other. In addition, it is found that, using the same forcing frequency $f = 18$ Hz and the same acceleration amplitude $A=17$ m/s², we can *not* observe any Faraday waves if there exists only the 8mm silicone oil (with the air) in the sealed Hele-Shaw cell, or if we increase the depth of pure ethanol up to 10mm. This phenomenon strongly suggests that the lower horizontally oscillating Faraday wave is excited by the upper vertically vibrating Faraday wave via the viscous friction on the interface between the two immiscible liquids.

The schematic illustration is as shown in figure 2(a), where H_1 and H_2 denote the wave height of the upper and lower Faraday waves, L denotes their wave length, δ is the distance between the two adjoining crests of the lower Faraday wave, respectively. In the case of $f = 18$ Hz and $A = 17$ m/s², the time-dependent variation of δ is as shown in figure 2(b), which can be fitted by a simple formula

$$\delta(t) = \frac{1}{2}(\delta_{max} + \delta_{min}) - \frac{1}{2}(\delta_{max} - \delta_{min}) \cos(\pi ft) \quad (1)$$

with $\delta_{max} = 14.08$ mm and $\delta_{min} = 7.26$ mm in a good agreement with the measured data, and by the fitted formula

$$\delta(t) = \frac{1}{2}(\delta_{max} + \delta_{min}) - \frac{1}{2}(\delta_{max} - \delta_{min}) \cos(\pi ft) + B \sin(\pi ft) \quad (2)$$

with $B = 0.4865$ mm in a better agreement.

With f fixed at 18Hz, the system of the two coupled Faraday waves can be observed within a region of the acceleration amplitude $12.5 \text{ m/s}^2 \leq A \leq$

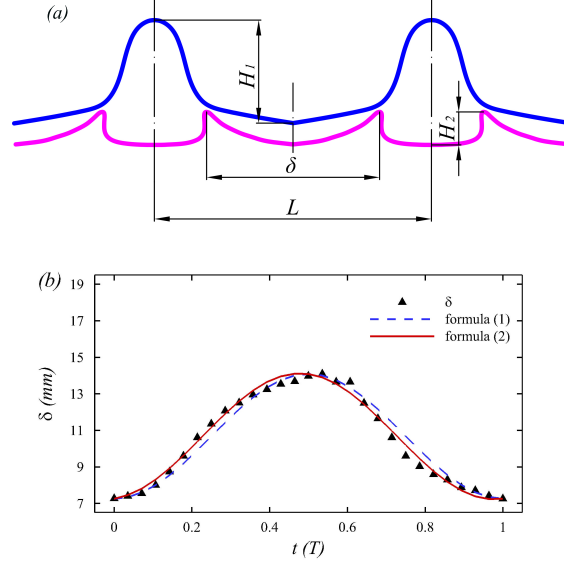


Figure 2: (Colour online) (a) Schematic illustration of the upper and lower Faraday waves. H_1 and H_2 denote the wave height of the upper and lower Faraday waves, L is their wave length, δ is the distance of the two crests of the lower Faraday wave, respectively. (b) Variation of $\delta(t)$ in case of $f = 18$ Hz and $A = 17$ m/s².

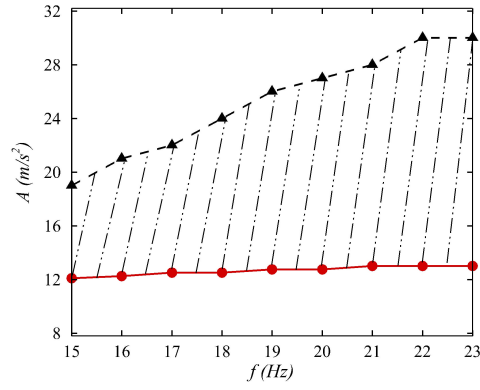


Figure 3: (Colour online) Existence window of A versus f for the couple two Faraday waves. Solid line: the lower threshold; Dashed line: the upper threshold.

24 m/s². When $A < 12.5$ m/s², no interfacial waves were observed at all. When $A > 24$ m/s², the interfacial waves at the upper and lower interface are disordered. In the cases of $f = 23$ Hz and $f = 15$ Hz, such kind of two coupled Faraday waves are always observed, but with different upper and lower thresholds of A . It is found that there exists the corresponding upper and lower thresholds of A for a given forcing frequency f , as shown in figure 3. Note that the lower threshold of A increases with the frequency f very slowly, but the upper threshold rises rapidly.

In the case of the fixed acceleration amplitude $A = 15$ m/s² with the different forcing frequency f in the region of $15 \text{ Hz} \leq f \leq 23 \text{ Hz}$, the wave height H_2 of the lower Faraday wave is a parabolic function of the wave height H_1 of the upper one, but δ_{max} has a linear relationship with the wave length L , respectively, as shown in figure 4(a,b). In the case of the fixed forcing frequency $f = 18$ Hz with the different acceleration amplitude A in the region of $14 \text{ m/s}^2 \leq A \leq 22 \text{ m/s}^2$, the wave heights H_2 of the lower Faraday wave has a linear relationship with the wave height H_1 of the upper one, but δ_{max} is a parabolic function of the wave length L , respectively, as shown in figure 4(c,d). These phenomena reveal the close relationship and strong coupling between the upper and lower Faraday waves.

Note that the upper Faraday wave looks like the traditional Faraday wave at the interface of two immiscible fluids only (such as water and air). For the sake of comparison, we measured the traditional Faraday wave at the interface of the air and pure ethanol with the same depth of 4mm (but *without* silicon oil) in the same Hele-Shaw cell [19, 20]. In the case of the fixed acceleration amplitude $A = 15$ m/s² with the different forcing frequency f in the region of $15 \text{ Hz} \leq f \leq 23 \text{ Hz}$, both of the wave height H_1 and wave length L of the upper Faraday wave decreases with the increase of the frequency f , as shown in figure 5(a,b). Besides, it is found that the wave height H_1 of the upper Faraday wave is always smaller than the wave height H_0 of the traditional one, but the wave length L of the upper Faraday wave is almost the same as that of the traditional one, respectively. This is easy to understand, since the upper layer liquid (pure ethanol) transfers some kinetic energy to the lower one (silicon oil) via viscous friction at their interface. In the case of the fixed frequency $f = 18$ Hz with the different acceleration amplitude A in the region $14 \text{ m/s}^2 \leq A \leq 22 \text{ m/s}^2$, both of the wave height H_1 and wave length L of the upper Faraday wave increases with the increase of A , as shown in figure 5(c,d). However, it is interesting that the traditional Faraday wave does not exist when $A > 17$ m/s², but

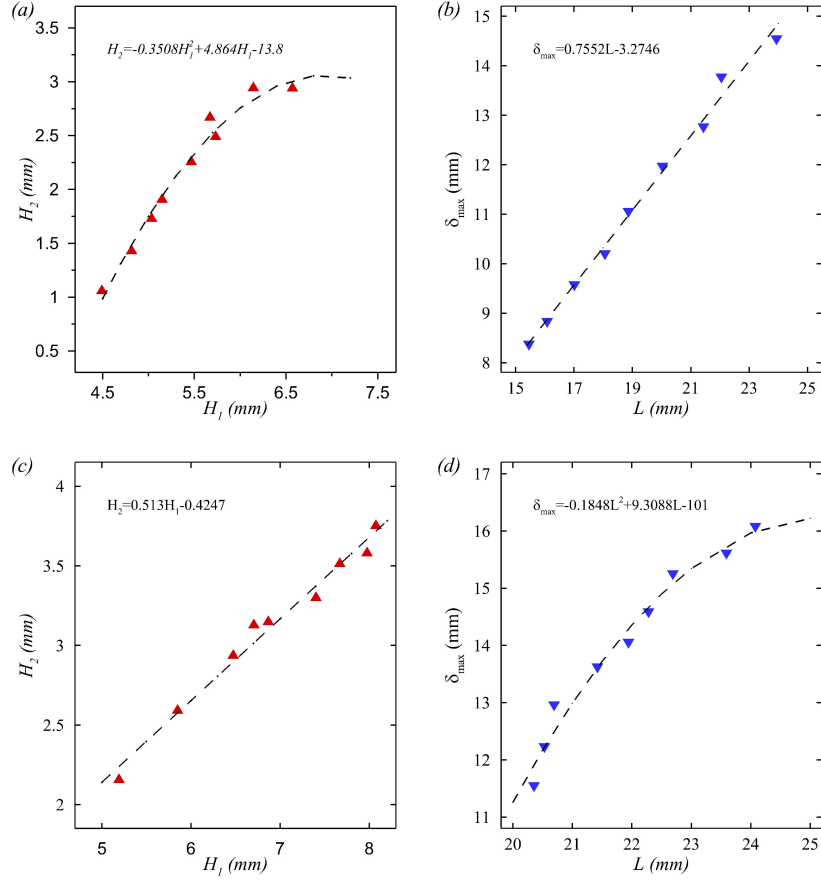


Figure 4: (Colour online) Relations between H_1 and H_2 , L and δ_{\max} of the coupled two Faraday waves. (a) and (b): in case of $A=15$ m/s² and 15 Hz $\leq f \leq 23$ Hz; (c) and (d): in the case of $f=18$ Hz and 14 m/s² $\leq A \leq 22$ m/s²; Dashed-line: the fitting formulas.

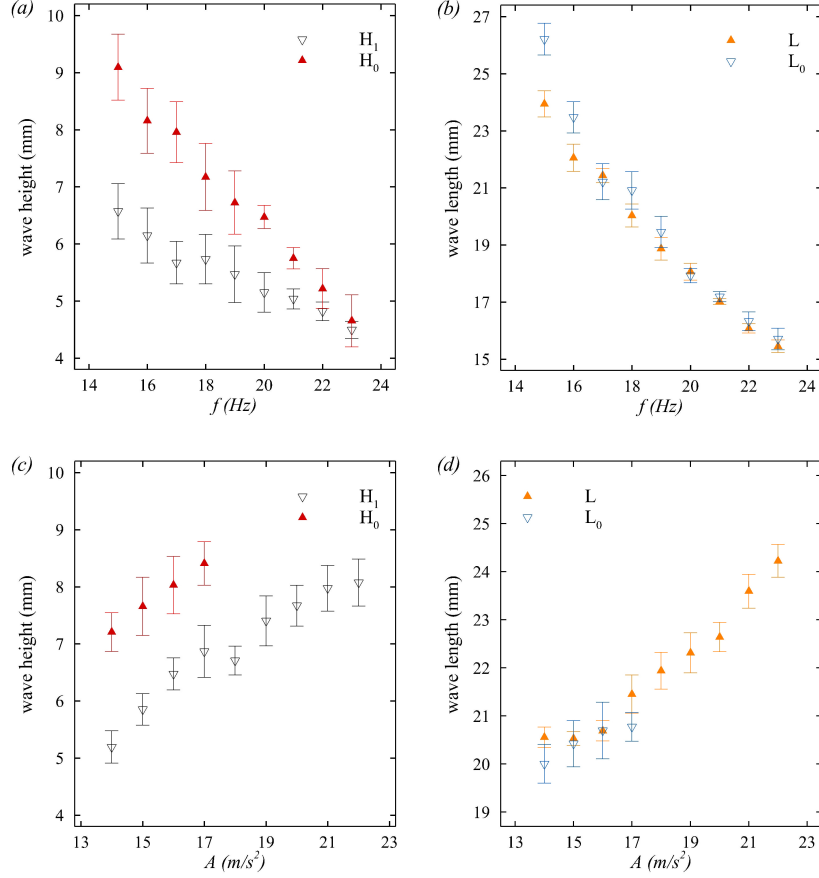


Figure 5: (Colour online) Comparison of the upper Faraday wave and the traditional Faraday wave of pure ethanol with the same physical parameters (but without the layer of silicon oil below), where H and L denote wave height and wave length of the upper Faraday wave, H_0 and L_0 denote those of the traditional one. (a) and (b): in case of $A=15 \text{ m/s}^2$ and $15 \text{ Hz} \leq f \leq 23 \text{ Hz}$; (c) and (d): in the case of $f=18 \text{ Hz}$ and $14 \text{ m/s}^2 \leq A \leq 22 \text{ m/s}^2$.

the system of the two coupled Faraday waves exists even for the acceleration amplitude A up to 22 m/s^2 . It means that, for a given forcing frequency f , the upper threshold of A for the coexistence of the two coupled Faraday waves is larger than that for only one traditional Faraday wave. It indicates that the system of the two coupled Faraday waves is stable even for a large acceleration amplitude A that corresponds to a high nonlinearity. Note that, for a given frequency f , more kinetic energy is needed to excite the system of the two coupled Faraday waves than the only one traditional Faraday wave. In addition, the wave length L of the upper Faraday wave is almost the same as L_0 of the traditional one, although its wave height H_1 is always smaller than H_0 . Furthermore, unlike the traditional Faraday wave, the upper Faraday wave temporarily loses its smoothness at around $t = T/4$ and $3T/4$, as shown in figure 1. All of these indicate that the upper Faraday wave is fundamentally different from the traditional one, although both of them are vertically vibrating waves. Finally, it should be emphasized that the upper and lower Faraday waves coexist and are strongly coupled: neither of them can exist along.

Note that Potosky and Bestehorn [15] numerically investigated the linear instability of Faraday waves of the three-layer fluids (air and two immiscible liquids) in a *three-dimensional* domain. However, they only gained the coupled two Faraday waves that vibrate vertically. Unlike their theoretical investigation, our physical experiments are related to the *two-dimensional* Faraday waves by means of physical parameters quite different from theirs: the ratio of the viscosity $\mu_2/\mu_1 \approx 318.2$ in our experiment is about 22 times larger than that considered by Potosky and Bestehorn [15].

4. Conclusions

In conclusion, we experimentally observed a system of the two-dimensional, two coupled Faraday waves at two interfaces of three layers of fluids (air, pure ethanol and silicon oil) in a sealed Hele-Shaw cell with periodic vertical vibration. The upper Faraday wave vibrates vertically, and the lower oscillates horizontally. They coexist and are strongly coupled. This system of two coupled Faraday waves has never been reported, to the best of our knowledge. So, it fleshes out the picture of Faraday waves as a type of vertical standing waves. They also bring us some new challenges in theoretical analysis and numerical simulations.

Acknowledgement

This work is partly supported by National Natural Science Foundation of China (Approval No. 11272209 and 11432009).

References

References

- [1] M. Faraday, On a peculiar class of acoustical figures; and on certain forms assumed by groups of particles upon vibrating elastic surfaces, *Philosophical transactions of the Royal Society of London* 121 (1831) 299–340.
- [2] T. B. Benjamin, F. Ursell, The stability of the plane free surface of a liquid in vertical periodic motion, in: *Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences*, Vol. 225, The Royal Society, 1954, pp. 505–515.
- [3] D. Binks, M.-T. Westra, W. van de Water, Effect of depth on the pattern formation of Faraday waves, *Physical review letters* 79 (25) (1997) 5010.
- [4] J. Rajchenbach, D. Clamond, A. Leroux, Observation of star-shaped surface gravity waves, *Physical review letters* 110 (9) (2013) 094502.
- [5] J. Bouchgl, S. Aniss, M. Souhar, Interfacial instability of two superimposed immiscible viscous fluids in a vertical Hele-Shaw cell under horizontal periodic oscillations, *Physical Review E* 88 (2) (2013) 023027.
- [6] G. Mercer, A. Roberts, Standing waves in deep water: Their stability and extreme form, *Physics of Fluids A: Fluid Dynamics* 4 (2) (1992) 259–269.
- [7] K. Kumar, L. S. Tuckerman, Parametric instability of the interface between two fluids, *Journal of Fluid Mechanics* 279 (1994) 49–68.
- [8] J. Wright, S. Yon, C. Pozrikidis, Numerical studies of two-dimensional Faraday oscillations of inviscid fluids, *Journal of Fluid Mechanics* 402 (2000) 1–32.

- [9] K. Takagi, T. Matsumoto, Numerical simulation of two-dimensional Faraday waves with phase-field modelling, *Journal of Fluid Mechanics* 686 (2011) 409–425.
- [10] D. Hill, The Faraday resonance of interfacial waves in weakly viscous fluids, *Physics of Fluids* 14 (1) (2002) 158–169.
- [11] A. Kityk, J. Embs, V. Mekhonoshin, C. Wagner, Spatiotemporal characterization of interfacial Faraday waves by means of a light absorption technique, *Physical Review E* 72 (3) (2005) 036209.
- [12] G. Pucci, E. Fort, M. B. Amar, Y. Couder, Mutual adaptation of a Faraday instability pattern with its flexible boundaries in floating fluid drops, *Physical review letters* 106 (2) (2011) 024503.
- [13] G. Pucci, M. B. Amar, Y. Couder, Faraday instability in floating liquid lenses: the spontaneous mutual adaptation due to radiation pressure, *Journal of Fluid Mechanics* 725 (2013) 402–427.
- [14] Y. Couder, S. Protiere, E. Fort, A. Boudaoud, Dynamical phenomena: Walking and orbiting droplets, *Nature* 437 (7056) (2005) 208–208.
- [15] A. Pototsky, M. Bestehorn, Faraday instability of a two-layer liquid film with a free upper surface, *Phys. Rev. Fluids* 1 (2016) 023901.
- [16] S. Amiroudine, F. Zoueshtiagh, R. Narayanan, Mixing generated by Faraday instability between miscible liquids, *Physical Review E* 85 (1) (2012) 016326.
- [17] S. Diwakar, F. Zoueshtiagh, S. Amiroudine, R. Narayanan, The Faraday instability in miscible fluid systems, *Physics of Fluids* 27 (8) (2015) 084111.
- [18] L. Chang, Y. Jian, J. Su, R. Na, Q. Liu, G. He, Nonlinear interfacial waves in a circular cylindrical container subjected to a vertical excitation, *Wave Motion* 51 (5) (2014) 804–817.
- [19] X. Li, Z. Yu, S. Liao, Observation of two-dimensional Faraday waves in extremely shallow depth, *Physical Review E* 92 (3) (2015) 033014.

- [20] X. Li, X. Li, S. Liao, Pattern transition of two-dimensional Faraday waves at an extremely shallow depth, *Sci. China-Phys. Mech. Astron.* 59 (11) (2016) 114712.